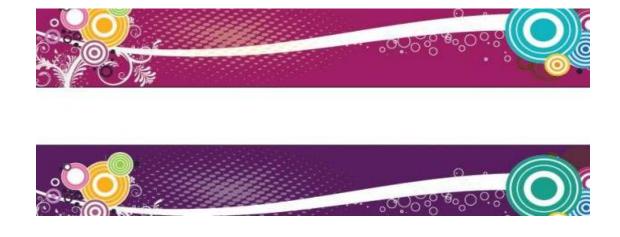
## A two thousand 9 after decimal point

Prove that the first two thousand digits after the decimal point in the value of  $(6 + \sqrt{35})^{2015}$  are all 9's.

Hint : If the given number is a man, find his wife.



Let 
$$M = (6 + \sqrt{35})^{2015}$$
,  $W = (6 - \sqrt{35})^{2015}$   
Let  $C = (6 + \sqrt{35})^{2015} + (6 - \sqrt{35})^{2015}$   
 $= 2\{6^{2015} + C_2^{2015}(6^{2013})(35^2) + C_4^{2015}(6^{2011})(35^4) + \dots + C_{2014}^{2015}(6)(35^{2014})\}$   
 $= 2k$ , an even integer

$$0 < 6 - \sqrt{35} = 0.0832 \dots < 0.1 = 10^{-1}$$

Hence  $0 < (6 - \sqrt{35})^{2015} < 10^{-2015}$ 

$$\therefore \ \left(6 + \sqrt{35}\right)^{2015} = 2k - \left(6 - \sqrt{35}\right)^{2015}$$

which is an integer short of 0.00...01 (2014 zeros after the decimal point)  $\therefore (6 + \sqrt{35})^{2015}$  has at least 2014 number of 9's after the decimal point.

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