A two thousand 9 after decimal point

Prove that the first two thousand digits after the decimal point in the value of $\left(6+\sqrt{35}\right)^{2015}$ are all 9's.

Hint : If the given number is a man, find his wife.

Let M = $\left(6+\sqrt{35}\right)^{2015}$, W = $\left(6-\sqrt{35}\right)^{2015}$

Let C $=\left(6+\sqrt{35}\right)^{2015}+\left(6-\sqrt{35}\right)^{2015}$

$= 2\{6^{2015}+C_2^{2015}(6^{2013})(35^2)+C_4^{2015}(6^{2011})(35^4)+\cdots+C_{2014}^{2015}(6)(35^{2014})\}$

= 2k , an even integer

But $0<6-\sqrt{35}=0.0832\ldots<0.1=10^{-1}$

Hence $0<\left(6-\sqrt{35}\right)^{2015}<10^{-2015}$

$\therefore\ \left(6+\sqrt{35}\right)^{2015}=2k-\left(6-\sqrt{35}\right)^{2015}$

which is an integer short of 0.00…01 (2014 zeros after the decimal point)

$\therefore\ \left(6+\sqrt{35}\right)^{2015}$ has at least 2014 number of 9's after the decimal point.

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